# Some results on the generalized comaximal graph of a near-ring

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**Abstract**

In [5], Sharma and Bhatwadekar introduced the notion of comaximal graph $\Gamma(R)$ on a commutative ring $R$ with identity taking elements of $R$ as vertices, where two distinct vertices $x$ and $y$ are adjacent if and only if $Rx + Ry = R$ and showed that $R$ is a finite ring if and only if the chromatic number of $G(R)$ is finite. With this definition, Maimani et al.[3], Wang[6] characterized several graph-theoretic properties such as connectedness, completeness, diameter, bipartiteness, etc. Later, in [2], Biswas et al. generalized the concept of comaximal graph of a commutative ring with identity and defined a graph $G(R)$ whose vertices are the elements of $R$ and any two distinct elements $a$ and $b$ are adjacent if and only if $aR + bR = eR$ for some non-zero idempotent $e$ in $R$ and studied the completeness of the graph $G(R)$ through the regular property of $R$. In [1], the authors mainly studied the genus, energy, and Hamiltonian property of $G(R)$.

Here, we extend the above concept in the setting of a near-ring as follows: We assign an undirected simple graph $\Gamma_G(N)$ on a near-ring $N$ with vertices as elements of $N$, where two distinct vertices $a$ and $b$ are adjacent if and only if $Na + Nb = Ne$ for some idempotent $e$ of $N$. Here, $Na$ is an $N$-subgroup of $N^N$, but it may not be a principal left ideal of $N$ as in the case of a commutative ring with identity. It is clear that if $N$ is a commutative ring with identity, then $\Gamma_G(N)$ is identical to the graph defined by Biswas et al.[2]. For convenience, we call $\Gamma_G(N)$ generalized comaximal graph on a near-ring $N$.

The following result can be found in [[4], Theorem 9.156, $p$. 346].

**Theorem 1.** *Let $N$ be a near-ring with identity. Then $N$ is regular if and only if for each $n \in N$, $Nn = Ne$ for some idempotent $e$ of $N$.*

With the help of the above result, we prove that if $N$ is a near-ring with identity such that all idempotents are central, then $\Gamma_G(N)$ is complete if and only if $N$ is regular. Also, we prove that if $N$ is a zero-symmetric reduced near-ring with identity, then $\Gamma_G(N)$ is complete if and only if $Na$ is a direct summand of $N$ for all $a \in N$. Moreover, we establish a relation among the completeness of $\Gamma_G(N)$, the regularity of $N$, the strongly regularity of $N$, and several radicals of $N$, for a particular class of near-rings $N$. Finally, for a finite zero-symmetric IFP near-ring $N$, we show that if $N$ is a unit regular or strongly regular or left duo regular or reduced left-morphic near-ring, then the numbers of regular elements and left-morphic elements of $N$ are equal to the clique number of $\Gamma_G(N)$.

**Key Words and Phrases:** Comaximal graph, regular near-ring, strongly regular near-ring, left-morphic near-ring.
**2020 Mathematics Subject Classification:** $05C25, 16Y30$.
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